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Quality of Schooling: Child Quantity-Quality Tradeoff, Technological Progress and Economic Growth

Swati Saini¹

Abstract

An overlapping generations version of an R&D-based growth model à la Diamond (1965) and Jones (1995) is built to examine how improvement in quality of schooling impact technical progress and long-run economic growth of an economy by influencing fertility and education decisions at household level. The results indicate that improvement in schooling quality triggers a child quantity-quality trade-off at household level when quality of schooling exceeds an endogenously determined threshold. At the household level, parents invest more in education of children and have lesser number of children in response to improvement in quality of schooling. This micro-level tradeoff has two opposing effects on aggregate human capital accumulation at macro level. Higher investment in education of a child stimulates the accumulation of human capital which fosters technical progress but the simultaneous decline in fertility rate reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The first effect prevails over latter only when quality of schooling is higher than the threshold.

JEL classification : I25, J13, J11, O31

Keywords : Human capital, fertility, quality of schooling, economic growth, innovation, imitation, demographic transition

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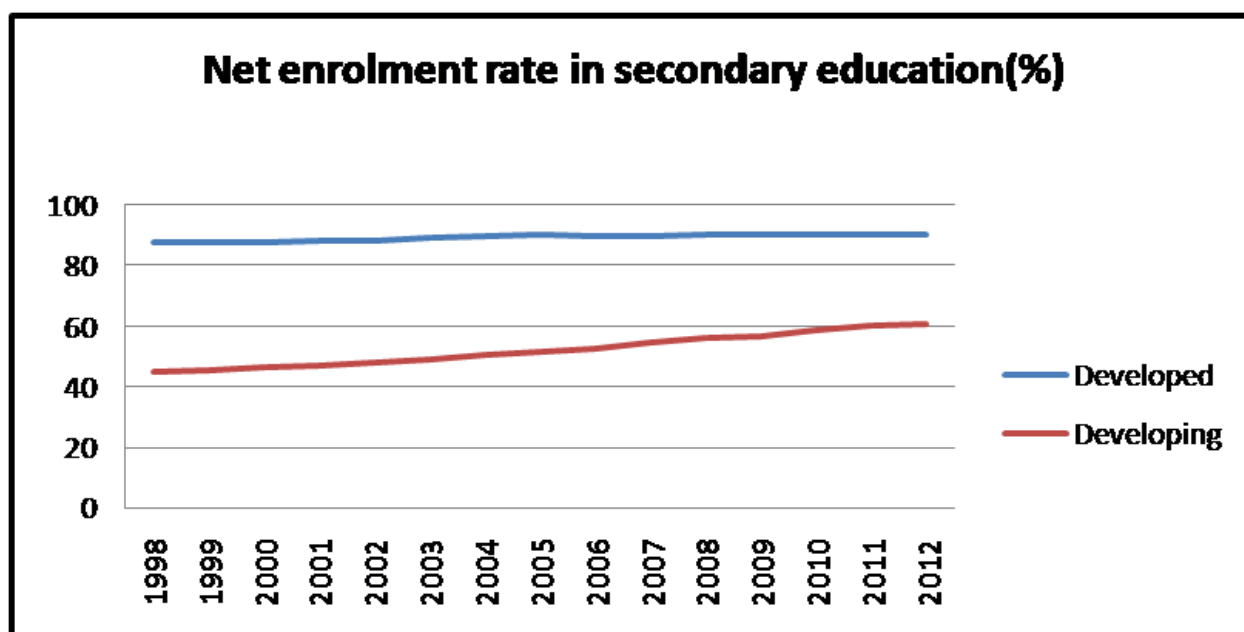
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1 Introduction

Human capital as a potential driver of technical change has emerged as an important determinant of economic growth in the endogenous growth literature. The policy emphasis on schooling in the development strategies of most countries mirrors the emphasis of research on the role of human capital in growth and development. Developing countries have focussed on improving the access to education so that their stock of human capital can be built up which, in turn, can be fruitfully employed to speed up the process of technological progress and diffusion and, therefore, spur economic growth. Developing countries have made considerable progress in closing the gap with developed countries in terms of school attainment (Figure 1). As can be seen, in 1998, the educational attainment gap (as measured by net enrolment

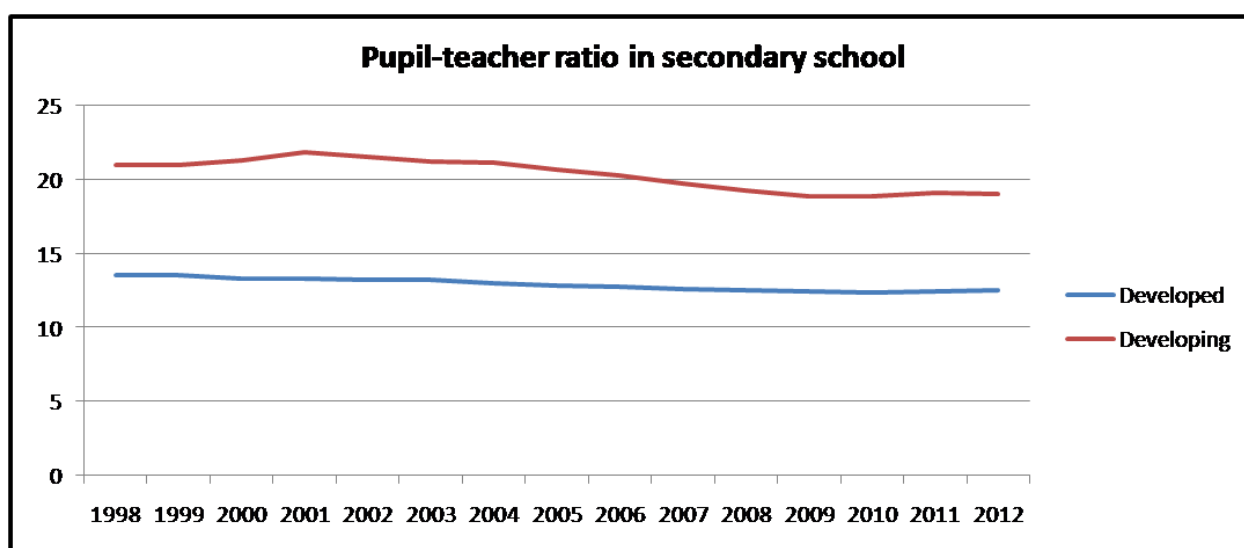
Figure 1: Comparison of secondary education attainment in developed and developing countries



rate in secondary schooling) between developed and developing countries was approximately 43%. In 2012, this educational attainment gap between the two country-groups narrowed down to 30%. Therefore, it can be said that developing countries have succeeded in narrowing the human capital gap with developed countries. However, the quality of human capital stock also matters for economic growth. This data on school attainment may be misleading without consideration of how much students are learning. Differences in economic growth across countries are closely related to cognitive skills as measured by achievement on international assessments of mathematics and science (Hanushek 2013). Hanushek and Kimko (2000) and Hanushek and Woessmann (2012) provide an extensive discussion of how scores from cognitive skill tests can be used to measure the quality of human capital and its effects

on economic growth. They use data from six voluntary international tests of mathematics and science to build a measure of quality of education. They find that the estimate of human capital quality has significantly positive impact on growth. Several studies have since found very similar results (see e.g. Bosworth and Collins (2003), Ciccone and Papaioannou (2009) Islam et. al. (2014)). This underscores the importance of cognitive skills for economic growth and, therefore, shifts attention to issues of school quality. In this respect, developing countries have been much less successful in closing the gap with developed countries. If we take pupil-teacher ratio as a proxy for quality of schooling, then Figure 1.2 reveals that the gap between developed and developing countries in terms of quality of schooling has remained more or less the same during 1998-2012. This analysis reveals that the schools

Figure 2: Gap in terms of quality of schooling



across diverse countries are not imparting the same amount of learning outcomes per year in all. If developing countries want to close the economic gap with the developed countries, then they need to focus on the quality of human capital stock as well.

Motivated by these observations and the empirical findings of Hanushek and Woessmann (2012), this paper intends to analyse how quality of schooling influences growth prospects of a country

The existing literature on quality of schooling and economic growth shows that economic growth and quality of schooling are positively correlated (see e.g. Bosworth and Collins (2003), Ciccone and Papaioannou (2009) Islam et. al. (2014)). This implies that developing countries should adopt a two-pronged approach to enhance the skill set of its workers. Under this approach, countries should focus on improving quality of education also along with improving access to education. Many existing studies (eg. Tamura (2001), Gilpin and Kaganovich (2012), Das and Guha (2012)) on the quality of schooling and economic growth focus on explaining how determinants of quality of schooling such as teacher-student ratio and teacher quality together impact the learning process and the consequent human capital formation

and therefore, economic growth. However, most of these studies assume exogenously determined population growth and therefore do not analyze the impact of quality on schooling on the tradeoff between quantity of and quality (education) of children (Beckerian tradeoff) at family level and its consequent impact on technical progress and economic growth.

Through this paper, we intend to go beyond existing literature by specifically modeling the relation between quality of schooling, child quantity-quality choice and its implication on technology and therefore growth. We build an overlapping generations version of an R&D-based growth model ‘a la Diamond (1965) and Jones (1995) to examine how improvement in quality of schooling impact technical progress and long-run economic growth of an economy by influencing fertility and education decisions at household level.

This paper is organized as follows. Section 2 discusses the basic structure of the model. Section 3 contains the key analytical results for a decentralized economy which provide the key propositions of this study. Section 4 concludes this study.

2 The Model

2.1 The Economic Environment

Consider a model economy populated by overlapping generations of people who live for two periods : adulthood and old age. Time is discrete and goes from 0 to ∞ . During childhood, individuals are reared and educated by their parents. All decisions are made at beginning of adulthood. Adults are identical in all aspects. They inelastically supply their skills on the labor market. Adults care about consumption, number and human capital of their children. During old age, individuals consume their savings plus interest. Abstracting from gender differences, each household has single parent. For avoiding indivisibility problem, we assume children are in continuous number. All individuals survive up to adulthood. Education of current period’s children determines human capital endowment of next period’s adult generation. Akin to Castello-Climent (2012), human capital accumulation function depends upon exogenously given quality of education system, parental investment in education and human capital of parent. Parental investment in education is the fraction of income spent on education of each child.

Individuals derive utility from $c_{1,t}$, their own consumption during adulthood; $c_{2,t+1}$, their own consumption during old age; n_t , number of children and h_{t+1} , human capital of children. Parents’ motivation to invest in human capital of children by spending on children’s education is driven by a “warm glow” of giving (Andreoni 1989) or preference for having “higher-quality” children (Becker 1960). The lifetime expected utility of individuals in generation t is given by :

$$u_t = \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1}n_t) \quad (1)$$

where positive weights β_1 and β_2 measure the importance of future consumption and child quantity and quality relative to current consumption in the utility function. Alternatively, following De la Croix and Doepke (2004), β_2 can be interpreted as altruism factor.

An adult's human capital is denoted by h_t and the wage per unit of human capital is w_t . Young adults spend their income on current consumption, savings for old-age consumption and child's education expenditure. Rearing a child necessarily takes fraction $\tau \in (0,1)$ of an adult's time. The budget constraints for the young and old adults are given by :

$$w_t h_t (1 - \tau n_t) = c_{1,t} + s_t + e_t (w_t h_t) n_t \quad (2)$$

$$c_{2,t+1} = R_{t+1} s_t \quad (3)$$

where e_t is the fraction of income per child spent on education, s_t is savings and R_{t+1} is gross interest rate. Assuming there is full depreciation of capital over the course of one generation, gross interest rate is equivalent to rental rate of capital which is given by $(1 + r_{t+1})$, r_{t+1} being the net interest rate. Non-negativity constraints apply to all variables.

The human capital of children h_{t+1} depends on human capital of parents h_t , parental investment in education per child e_t and quality of education system θ which is exogenously given.

$$h_{t+1} = (\mu + \theta e_t)^\epsilon h_t, \quad \epsilon < 1 \quad (4)$$

The parameters satisfy $\mu > 0$ and $\epsilon \in (0,1)$. ϵ measures the returns to education. Following Strulik et al (2013), μ can be considered as basic skills learnt by children by observing and imitating parents. Presence of μ ensures that human capital remains positive even if parents do not invest in education. The assumption that quality of schooling is an argument in human capital accumulation function is consistent with a number of studies. Hanushek et al (2008) find that lower-quality schools lead to higher dropout rates in Egyptian primary schools. Similarly, Hanushek and Woessmann (2008) find that cognitive skills, a proxy for educational quality is positively related to individual earnings.¹ Parental human capital h_t as an input in human capital accumulation technology represents intergenerational transfers of human capital which is a common assumption in the literature (see, e.g., de la Croix and Doepke (2004), Tamura (2001), Kalemli-Ozcan (2002, 2003)).

Individuals maximize utility (1) with respect to the constraints (2) - (4) using control variables $c_{1,t}$, s_t , n_t and e_t . The solution to individuals' decision problem can either be interior, or at a corner where the individuals choose zero education. The first-order conditions yield the following solution (5)-(6) for consumption and savings irrespective of whether education is interior or at the corner:²

$$c_{1,t} = \frac{w_t h_t}{1 + \beta_1 + \beta_2} \quad (5)$$

$$s_t = \frac{\beta_1 w_t h_t}{1 + \beta_1 + \beta_2} \quad (6)$$

¹See Hanushek et al (2011) for review of evidence on impact of various measures of schooling quality on learning and time in school.

²Detailed mathematical derivations are provided in Appendix A1.

For child quantity and quality, there exists a threshold level of quality of schooling. If quality of schooling falls below the threshold, adults do not spent on child quality and maximize child quantity. This constitutes the corner solution. In particular, following results are derived from first order conditions:

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon} \\ \frac{\tau\theta\epsilon - \mu}{\theta(1 - \epsilon)}, & \text{otherwise} \end{cases} \quad (7)$$

$$n_t = \begin{cases} \frac{\beta_2\epsilon\theta}{(1 + \beta_1 + \beta_2)\mu}, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon} \\ \frac{\beta_2\theta(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)}, & \text{otherwise} \end{cases} \quad (8)$$

Inserting (8) in (4), we get an equation of motion for human capital :

$$h_{t+1} = \begin{cases} \mu^\epsilon h_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon} \\ \left[\frac{\epsilon(\tau\theta - \mu)}{(1 - \epsilon)} \right]^\epsilon h_t, & \text{otherwise} \end{cases} \quad (9)$$

Below the threshold, quality of schooling is not an argument in human capital production function. Without education expenditure, human capital of next generation consists of basic skills only. From (6)-(9), irrespective of whether quality of schooling exceeds threshold or not, savings and consumption are increasing in $w_t h_t$ and there is no direct effect of income on fertility because a positive income effect of an increase in wages on fertility is balanced by a negative substitution effect. The quality of schooling has a direct bearing on child quantity and quality. The following lemma shows how quality of schooling influences fertility behaviour.

Lemma 1 *When quality of schooling is high enough to surpass the threshold, a marginal improvement in quality of schooling triggers a child quantity-quality tradeoff such that adults have less children and invest more in education per child in response to improvement in quality of schooling.*

Proof. To see the effect when quality of schooling is above the threshold, we take the derivatives of the interior solution of e_t and n_t with respect to θ in (7) and (8):

$$\frac{\partial n_t}{\partial \theta} = -\frac{\mu\beta_2(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^2} < 0$$

$$\frac{\partial e_t}{\partial \theta} = \frac{\mu}{(1 - \epsilon)\theta^2} > 0$$

■

Altogether, it can be seen that fertility changes are directly triggered by quality of schooling. Any improvement in quality of schooling over and above the threshold makes learning

in schools more effective and therefore, increases marginal returns to investment in human capital. Consequently, a parent reduces fertility and spends more on education per child. Thus, quality of schooling can be perceived as another plausible mechanism for triggering child quantity-quality tradeoff besides other commonly proposed mechanisms such as declining child mortality (Strulik 2004, Soares 2005), rise in life expectancy of parents (Boucekkine et al 2002, 2003; Kalemli-Ozcan 2002,2003; Hashimoto and Tabata 2016), technical progress (Galor and Weil 1999) and decline in gender wage gap (Galor and Weil 1996). These theoretical results are in line with recent empirical findings. For example, Hanushek et al (2008) find that lower quality of schooling leads to higher dropout rates in Egyptian primary schools. A cross-country analysis by Castello and Climent (2012) reveals that quality of education has a positive effect on enrollment rates in secondary schooling only when quality of schooling is sufficiently high.

Lemma 2 *An increase in returns to education, ϵ , leads to a child quantity-quality trade-off wherein parents educate their children and have less children when quality of schooling surpasses the threshold.*

Proof. Taking the derivatives of the interior solution of e_t and n_t with respect to ϵ in (7) and (8):

$$\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2 \theta}{(1 + \beta_1 + \beta_2)(\tau \theta - \mu)} < 0$$

$$\frac{\partial e_t}{\partial \epsilon} = \frac{\tau \theta - \mu}{\theta(1 - \epsilon)} > 0$$

■ This implies returns to education is another factor that can trigger a child quantity-quality trade-off. High returns to education implies education makes human capital more productive. Therefore, parents invest in education of their children and decide to have less number of children. However, when quality of schooling is less than the threshold, then parents decide not to make any investment in education of children and therefore, returns to schooling has no effect on child quality and child quantity is maximized.

2.2 Production Structure

The production structure of the economy closely follows Romer (1990) and Jones (1995). The economy consists of three sectors: final goods sector, intermediate goods sector and R&D sector. R&D sector employs human capital to produce blueprints of intermediate goods. Intermediate goods are produced by monopolistic firms using physical capital and intermediate good-specific blueprint. Final goods sector produces final good competitively using land and variety of intermediate goods as inputs.

2.2.1 Final Goods Sector

The final homogenous good (Y_t) is produced and sold in a competitive market. For any firm, the production structure at time t is defined as:

$$Y_t = l_t^{1-\alpha} \sum_{n=1}^{A_t} x_{it}^\alpha, \quad 0 < \alpha < 1 \quad (10)$$

The production of final good uses land and a variety of intermediate inputs. For simplicity, the total supply of land l_t has been normalized to 1. $x_{i,t}$ is the quantity of i th intermediate input that is used in final goods production and A_t is the number of available varieties of intermediate inputs or the level of technological knowledge that grows through R&D. The parameter α is the capital share in final goods production. The price of final good P_Y has been normalized to 1. In each period t , the final good producers solve the following profit maximization problem with respect to their choice of range of intermediate inputs :

$$\text{Max}_{x_{it}} \pi_t(Y) = l_t^{1-\alpha} \sum_{n=1}^{A_t} x_{it}^\alpha - \sum_{n=1}^{A_t} p_{it} x_{it} \quad (11)$$

where p_{it} is the unit monopoly price of i th intermediate input. The first order condition imply that :

$$p_{it} = \alpha l_t^{1-\alpha} x_{it}^{\alpha-1} \quad (12)$$

2.2.2 Intermediate Goods Sector

Each intermediate good i is produced by monopolist producer who holds the blueprint to produce x_{it} quantity at time t . Each intermediate good uses only capital in a one-to-one production technology, or $x_{it} = K_{it}$. Thus, the amount of intermediate inputs produced of all types equals the aggregate capital stock of the economy.

$$\sum_{n=1}^{A_t} x_{it} = K_t \quad (13)$$

Each i th intermediate good producer maximizes profits with respect to his/her choice of capital. That is,

$$\text{Max}_{x_{it}} \pi_t(i) = p_{it} x_{it} - r_t K_{it} = \alpha l_t^{1-\alpha} x_{it}^\alpha - r_t x_{it} \quad (14)$$

where the expression in R.H.S derives from substituting solution to p_{it} from eq (12) and $x_{it} = K_{it}$. r_t is price per unit of capital. The first order condition leads to

$$\alpha^2 l_t^{1-\alpha} x_{it}^{\alpha-1} = r_t \quad (15)$$

Using (12), we get the solution to equilibrium price as $p_{it} = p_t = \frac{r_t}{\alpha}$. This is the monopoly price charged as a markup over marginal cost. Note that being independent of i , it is constant across all intermediate goods. From eq (12), this implies that quantity produced of each i is same, that is $x_{it} = x_t = \left[\frac{\alpha^2}{r_t}\right]^{\frac{1}{1-\alpha}} l_t$. At equilibrium, the net profit of i th monopolist is given by

$$\pi_t = p_t x_t - r_t x_t \equiv \left[\frac{r_t}{\alpha} - r_t\right] x_t \equiv \left[\frac{1-\alpha}{\alpha}\right] r_t x_t \quad (16)$$

$$= \alpha(1-\alpha) l_t^{1-\alpha} x_t^\alpha. \quad (17)$$

where last expression has been derived using eq (15) and $x_{it} = x_t$ at equilibrium. Since at equilibrium, intermediate inputs are sold at the same price and demanded at equal quantities, aggregate physical capital is given by $K_t = A_t x_t$. Inserting this information into the production function of final goods, eq.(10) simplifies to

$$Y_t = l_t^{1-\alpha} A_t^{1-\alpha} K_t^\alpha. \quad (18)$$

Accordingly, equilibrium profits of i th monopolist in (17) simplifies to

$$\pi_t = \alpha(1-\alpha) \frac{Y_t}{A_t} \quad (19)$$

Also the price per unit of capital can be expressed as:

$$r_t = \alpha^2 l_t^{1-\alpha} \left[\frac{A_t}{K_t}\right]^{1-\alpha} \quad (20)$$

2.2.3 R&D Sector

The R&D sector produces blueprint of an intermediate variety by innovating upon the local technology level. Following Papageorgiou and Perez-Sebastian (2006), the production function of technology is postulated as:

$$A_{t+1} - A_t = \delta H_t^\lambda A_t^\phi \quad (21)$$

where $A_{t+1} - A_t$ are new blueprints which depend positively on number of already existing ideas, A_t and human capital employed in R&D sector, H_t . The parameter δ denotes general productivity in R&D. $0 \leq \phi < 1$ measures intertemporal knowledge spillovers (standing-on-shoulders effect) and $0 \leq \lambda < 1$ measures returns to R&D effort (stepping-on-toes effect). \bar{A}_t is world technology frontier that grows exogenously at rate $g_{\bar{A}}$.³

Firms in R&D sector maximize their profits

$$\pi_{t,A} = p_t^A (A_{t+1} - A_t) - w_t H_t \quad (22)$$

³The standing-on-shoulders effect may arise as existing knowledge contributes to the capacity to innovate. The stepping-on-toes effect may arise due to competition among multiple R&D firms to become the first to succeed at creating and patenting a new good or process. If all other factors are held constant, an increase in R&D efforts will induce increased duplication of research efforts leading to stepping-on-toes effect.

where p_t^A is price of a blueprint, $A_{t+1} - A_t$ are number of new blueprints discovered and w_t is the wage rate.

Using eq.(21), profit function is written as

$$\pi_{t,A} = p_t^A(\delta H_t^\lambda A_t^\phi) - w_t H_t \quad (23)$$

Maximization of profits leads to following optimality condition

$$w_t = \frac{\lambda p_t^A (A_{t+1} - A_t)}{H_t} \quad (24)$$

Akin to Strulik et al (2013), we assume that patent protection for a newly discovered blueprint lasts only for one period t (i.e. one generation). In the next period $(t+1)$, the patent right is randomly sold to somebody from currently active generation and the revenue, thus generated, is spent unproductively on public consumption. This assumption simplifies the exposition considerably as it keeps the basic incentive to create new knowledge intact while avoiding the intertemporal problems of patent pricing and patent holding. In the R&D sector, once a blueprint has been produced, a large number of potential intermediate input producers bid for the patent of the blueprint. The decision to produce a new intermediate variety depends on a comparison of operating profits that can be earned by producing an intermediate variety in time period t (when patent protection is valid) and the cost of buying blueprint. Since market for blueprints is competitive, price of blueprint will be bid up until it is equal to the operating profit of intermediate input firm in period t . Therefore, price of blueprints can be written as

$$p_t^A = \pi_t = \alpha(1 - \alpha) \frac{Y_t}{A_t} \quad (25)$$

which follows from eq (19). Accordingly, wage rate in eq.(24) can be expressed as

$$w_t = \lambda \alpha (1 - \alpha) \frac{Y_t}{H_t} g_{A,t} \quad (26)$$

where $g_{A,t} = \frac{A_{t+1} - A_t}{A_t}$.

3 Steady-State Analysis

This section examines the dynamic properties of our model economy. First, we discuss the dynamics of physical factors of production. The population N_t grows at the fertility rate.

$$N_{t+1} = n_t N_t \quad (27)$$

Taking child rearing time into account, the size of the workforce is given by $L_t = (1 - \tau n_t)N_t$. Since child rearing costs are constant and from (8), we know that fertility rate is also constant over time, therefore the workforce grows at the fertility rate.

$$L_{t+1} = n_t L_t \quad (28)$$

Assuming physical capital is to depreciate fully within a generation so that next period's capital stock consists of this period's aggregate savings, the market clearing condition for capital market is

$$K_{t+1} = s_t N_t \quad (29)$$

where N_t is the population of generation t . Further, inserting the solutions for savings from (6) and wages from (26) and (18) into (29) and using the fact that $N_t = L_t$ from (27) and (28), we get the equation governing the evolution of aggregate physical capital.

$$K_{t+1} = B_t K_t^\alpha A_t^{1-\alpha} g_{A,t} \quad (30)$$

where $B_t = \left[\frac{\beta_1}{1+\beta_1+\beta_2} \right] \lambda \alpha (1 - \alpha) l_t^{1-\alpha}$. Next we discuss the dynamics of aggregate human capital ($H_t \equiv h_t L_t$). The dynamics of per capita human capital are given by (9). Using (9) and (28), the equation for aggregate human capital accumulation can be written as

$$\frac{H_{t+1}}{H_t} = \begin{cases} \mu^\epsilon n_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon} \\ \left[\frac{\epsilon(\tau\theta - \mu)}{(1 - \epsilon)} \right]^\epsilon n_t, & \text{otherwise} \end{cases} \quad (31)$$

From (21), the dynamics of total factor productivity are expressed as

$$A_{t+1} = A_t + \delta H_t^\lambda A_t^\phi \quad (32)$$

This system of equations fully describes the equilibrium dynamics of our model economy. Note that these equations hold during the transition to the balanced growth path and along the balanced growth path itself. We consider only the case of an economy where quality of schooling surpasses the threshold and therefore, characterize the balanced growth path of this type of economy only.

3.1 Balanced growth path and its comparative statics

A balanced growth path (BGP) is defined as a steady state of the economy at which growth rate of a variable does not change over time. We begin with evaluating physical capital accumulation along the BGP. From eq.(30), we deduce

$$1 + g_{K,t} \equiv \frac{K_{t+1}}{K_t} = \left[\frac{K_t}{K_{t-1}} \right]^\alpha \left[\frac{A_t}{A_{t-1}} \right]^{1-\alpha} \left[\frac{g_{A,t}}{g_{A,t-1}} \right] \quad (33)$$

Using that at steady state, $\frac{K_{t+1}}{K_t} = \frac{K_t}{K_{t-1}}$ and $g_{A,t} = g_{A,t-1}$, we obtain

$$g_K = g_A \quad (34)$$

The growth of physical capital and productivity are positively correlated at steady state. Next, we consider growth rate of total factor productivity, we observe from (32) that

$$1 + g_{A,t} \equiv \frac{A_{t+1}}{A_t} = 1 + \frac{\delta^{\frac{1}{1-\phi}} H_t^{\frac{\lambda}{1-\phi}}}{A_t} \quad (35)$$

Using the definition of BGP, we derive the long run rate of technological progress under innovation regime as

$$(1 + g_A) = [(1 + g_h)n]^{\frac{\lambda}{1-\phi}} \quad (36)$$

The R.H.S follows from the definition of aggregate human capital $H_t = h_t L_t$ and (28)⁴ Next, we determine growth rates of aggregate GDP and per capita consumption along BGP. From (18), we observe

$$1 + g_{Y,t} \equiv \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t}^\alpha \frac{A_{t+1}}{A_t}^{1-\alpha} \quad (37)$$

Using (34), the long run growth rate of GDP is expressed as

$$g_Y = g_A \quad (38)$$

Putting together all information from (34), (36) and (38), we derive the balanced growth path of the economy.

$$g_K = g_Y = g_A = [(1 + g_h)n]^{\frac{\lambda}{1-\phi}} - 1 \quad (39)$$

where $[(1 + g_h)n] = (1 + g_H) = \frac{\beta_2 \theta \epsilon^\epsilon (1 - \epsilon)^{1-\epsilon}}{(1 + \beta_1 + \beta_2)(\tau \theta - \mu)^{1-\epsilon}}$. This follows after substituting value of n from (8) in (31).

From the consumer's optimisation exercise, we observe

$$\frac{c_{t+1}}{c_t} = \beta_1 (1 + r_{t+1}) \quad (40)$$

The R.H.S follows from substituting values of c_t and s_t from eq.(5) and (6) in (3). Using (20), we derive

$$\frac{c_{t+1}}{c_t} = \beta_1 \left[1 + \alpha^2 l_t^{1-\alpha} \left[\frac{A_t}{K_t} \right]^{1-\alpha} \right] \quad (41)$$

⁴We have dropped the time index of fertility rate as fertility rate remains constant over time.

Along BGP, since $g_K = g_A$, per capita consumption grows at a constant rate. Thus, an economy with a quality of schooling such that $\theta > \frac{\mu}{\tau\epsilon}$, converges towards a path of self-sustained constant economic growth in the long-run.

Proposition 1 *Aggregate output, physical capital stock, total factor productivity and per capita consumption grow at a constant rate along the balanced growth path characterised by (39) and (41).*

The self-sustained growth path is driven by human capital accumulation. At micro level, parents decide to have lesser number of children and invest more in their education. At the macro level, this tradeoff raises the rate of human capital accumulation which encourages faster technological progress and therefore, economic growth.

However, (39) suggest that technological progress and aggregate output are positively correlated with population growth. This implies that decline in population growth entails a decline in rate of technical progress as postulated by conventional R&D based growth models (Romer 1990, Jones 1995). This type of macro-level superficial examination misses the point that aggregate human capital accumulation and fertility rate are inversely related via quality-quantity tradeoff at family level as shown in Lemma 1. The investment in education increases and fertility rate falls simultaneously as the quality of schooling increases. This quality-quantity tradeoff implies that effect of population growth on total factor productivity growth and GDP growth cannot be analysed in isolation keeping human capital growth constant. This leads to the question how improvement in quality of schooling affects total factor productivity growth and therefore, economic growth by influencing fertility and education decisions.

Intuitively, quality of schooling has two opposing effects on human capital accumulation. An improvement in quality of schooling increases investment in education of a child. This stimulates the accumulation of human capital which fosters technical progress leading to a higher economic growth in the economy. This effect can be regarded as growth-stimulating effect. The increase in education is also accompanied by a decline in fertility rate as quality of education improves. This constitutes the growth-impeding effect that reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. Total factor productivity growth and economic growth will accelerate or decelerate depending upon relative magnitude of the two effects. To determine which effect will prevail in long-run, we take the derivative of the growth rate of aggregate human capital with respect to schooling quality. Detailed derivation is provided in Appendix A2.

$$\frac{\partial g_H}{\partial \theta} = \left[\frac{\theta \beta_2 \tau \epsilon^{1+\epsilon} (1-\epsilon)^{1-\epsilon}}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^{2-\epsilon}} - \frac{\mu \beta_2 \epsilon^\epsilon (1-\epsilon)^{1-\epsilon}}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^{2-\epsilon}} \right] \quad (42)$$

The first and second term in parenthesis represent the growth-stimulating and growth-impeding effects of quality of schooling on human capital accumulation respectively. It can be observed that the growth stimulating effect dominates growth-impeding effect when $\theta > \frac{\mu}{\tau\epsilon}$. As a result, growth rate of technology increases in response to an increase in schooling quality which sustains economic growth in the long-run.

Proposition 2 *The long-run rate of technical progress and economic growth increases in re-*

sponse to an improvement in quality of schooling when quality of schooling is high enough to surpass the threshold.

This result is similar to Hashimoto and Tabata(2016) finding about old-age survival probability and economic growth. They find that in economies in which old-age survival probability is sufficiently low, an increase in old-age survival probability motivates individuals to invest more in their own education, accelerating the accumulation of per capita human capital and thereby enhancing the long-run growth rate of the economy. However, in economies in which old-age survival probability is sufficiently high, an increase in old age survival probability will lead to declines in population growth rates, thereby lowering the long-run growth rate of the economy.

We next consider per capita income, $y_t = Y_t/L_t$. At steady state, its growth rate is given by:

$$g_y = (1 + g_h)^{\frac{\lambda}{1-\phi}} n^{\frac{\lambda+\phi-1}{1-\phi}} \quad (43)$$

If we differentiate per capita income with respect to θ and ϵ , we get⁵ :

$$\frac{\partial g_y}{\partial \theta} = \frac{1 + g_y}{1 - \phi} \left[\frac{\lambda \tau \epsilon}{\tau \theta - \mu} + \frac{\mu(1 - \lambda - \phi)}{\theta(\tau \theta - \mu)} \right] \quad (44)$$

$$\frac{\partial g_y}{\partial \epsilon} = \frac{\lambda(1 + g_y)}{1 - \phi} \left[1 + \log \frac{\epsilon}{1 - \epsilon} + \log(\tau \theta - \mu) + \frac{\epsilon}{1 - \epsilon} \right] + \frac{(1 + g_y)(1 - \lambda - \phi)}{(1 - \phi)(1 - \epsilon)} \quad (45)$$

An examination of these derivatives gives the following result.

Proposition 3 *Growth rate of per capita income is unambiguously increasing in quality of schooling and it is unambiguously increasing in returns to education only under the parametric restriction :*

$$\epsilon > 1/2 \& \lambda + \phi \leq 1$$

As we know from Lemma 1, improvement in quality of schooling triggers the quantity-quality trade-off leading to decrease in fertility rate and increase in education expenditure. At the macro level, this leads to a decline in population growth and therefore, increase in aggregate income and per capita income. This follows from Proposition 1. Qualitatively, the same argument of quantity-quality tradeoff holds true for returns to education from Lemma 2. This completes the description of how an economy having a good schooling system grows at a self-sustained rate in the long-run.

4 Conclusion

An overlapping generations version of an R&D-based growth model ‘a la Diamond (1965) and Jones (1995) is build to examine how improvement in quality of schooling impact technical progress and long-run economic growth of an economy. The quality of schooling triggers

⁵Detailed derivation is provided in Appendix A3

a child quantity-quality tradeoff at the micro level when quality of schooling surpasses an endogenously determined threshold. When quality of schooling surpasses the threshold, then parents invest in education of their children and have less number of children. However, parents focus on maximizing fertility and do not educate their children when quality of schooling is less than the threshold. This micro-level tradeoff has repercussions at the macro level. This micro-level tradeoff has two opposing effects on aggregate human capital accumulation at macro level. Higher investment in education of a child stimulates the accumulation of human capital which fosters technical progress but the simultaneous decline in fertility rate reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The first prevails over latter when quality of schooling is higher than the threshold. Accordingly, the economy is on a self-sustained growth path in the long-run when quality of schooling is higher than the threshold.

Appendix A1: Solution to Household's optimization exercise

The utility function is described as follows :
Maximize

$$\begin{aligned} u_t &= \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1}n_t) \\ &\text{subject to} \\ w_t h_t (1 - \tau n_t) &= c_{1,t} + s_t + e_t (w_t h_t) n_t \\ c_{2,t+1} &= R_{t+1} s_t \\ h_{t+1} &= (\mu + \theta e_t)^\epsilon h_t, \quad \epsilon < 1 \end{aligned}$$

After substituting for $c_{2,t+1}$ and h_{t+1} , the langragean for this problem is formulated as :

$$\begin{aligned} L &= \log c_{1,t} + \beta_1 \log(R_{t+1} s_t) + \beta_2 \log n_t + \beta_2 \epsilon \log(\mu + \theta e_t) + \beta_2 \epsilon \log h_t \\ &\quad + \phi [w_t h_t (1 - \tau n_t) - c_{1,t} - s_t - e_t n_t (w_t h_t)] \end{aligned}$$

The choice variables are $c_{1,t}$, s_t, e_t and n_t . The first order conditions are :

$$\frac{\partial L}{\partial c_{1,t}} = 0 \Leftrightarrow \frac{1}{c_{1,t}} - \phi = 0 \Leftrightarrow c_{1,t} = \frac{1}{\phi} \quad (\text{A1})$$

$$\frac{\partial L}{\partial s_t} = 0 \Leftrightarrow \frac{\beta_1}{s_t} - \phi = 0 \Leftrightarrow s_t = \frac{\beta_1}{\phi} \quad (\text{A2})$$

$$\frac{\partial L}{\partial n_t} = 0 \Leftrightarrow \frac{\beta_2}{n_t} - \phi \tau w_t h_t - \phi e_t w_t h_t = 0 \Leftrightarrow \frac{\beta_2}{n_t} = \phi [\tau + e_t] w_t h_t \Leftrightarrow n_t = \frac{\beta_2}{\phi [\tau + e_t] w_t h_t} \quad (\text{A3})$$

$$\frac{\partial L}{\partial e_t} = 0 \Leftrightarrow \frac{\beta_2 \epsilon \theta}{\mu + \theta e_t} - \phi n_t w_t h_t = 0 \Leftrightarrow n_t = \frac{\beta_2 \epsilon \theta}{\phi [\mu + \theta e_t] w_t h_t} \quad (\text{A4})$$

From eq A3 and A4, the L.H.S can be equated to yield

$$\mu + \theta e_t = \epsilon \theta [\tau + e_t] \Leftrightarrow \mu - \epsilon \theta \tau = e_t \theta [\epsilon - 1]$$

$$e_t = \frac{\mu - \epsilon \theta \tau}{\theta(\epsilon - 1)} = \frac{\epsilon \theta \tau - \mu}{\theta(1 - \epsilon)}$$

Hence, we have

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau \epsilon} \\ \frac{\tau \theta \epsilon - \mu}{\theta(1 - \epsilon)}, & \text{otherwise} \end{cases} \quad (\text{A5})$$

Next, we know the budget constraint is given by:

$$w_t h_t (1 - \tau n_t) = c_{1,t} + s_t + e_t (w_t h_t) n_t$$

From eqA3, $e_t n_t (w_t h_t)$ can be expressed as

$$e_t n_t (w_t h_t) = \frac{\beta_2}{\phi} - \tau n_t w_t h_t$$

and for c_t & s_t from eq A1 and A3, the budget constraint can be expressed as

$$w_t h_t - \tau n_t w_t h_t = \frac{1}{\phi} + \frac{\beta_1}{\phi} + \frac{\beta_2}{\phi} - \tau n_t w_t h_t$$

which on simplifying, leads to

$$\phi = \frac{1 + \beta_1 + \beta_2}{w_t h_t} \quad (\text{A6})$$

whose substitution into eqA1 and A3 yields,

$$c_{1,t} = \frac{w_t h_t}{1 + \beta_1 + \beta_2} \quad (\text{A7})$$

$$s_t = \frac{\beta_1 w_t h_t}{1 + \beta_1 + \beta_2} \quad (\text{A8})$$

Substituting for e_t from eqA5 and for ϕ from eq A6, yields

$$n_t = \begin{cases} \frac{\beta_2 \epsilon \theta}{(1 + \beta_1 + \beta_2) \mu}, & \text{if } \theta \leq \frac{\mu}{\tau \epsilon} \\ \frac{\beta_2 \theta (1 - \epsilon)}{(1 + \beta_1 + \beta_2) (\tau \theta - \mu)}, & \text{otherwise} \end{cases} \quad (\text{A9})$$

This completes the solution to the utility maximization exercise of households.

Appendix A2: Proof of Proposition 2

We know that $(1 + g_H) = (1 + g_h).n$ Differentiating both the sides w.r.t θ , we get

$$\frac{\partial g_H}{\partial \theta} = (1 + g_h) \frac{\partial n}{\partial \theta} + n \frac{\partial g_h}{\partial \theta} \quad (\text{A10})$$

From Lemma 1, we have

$$\frac{\partial n_t}{\partial \theta} = -\frac{\mu\beta_2(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^2}$$

and it is given $(1 + g_h) = \left[\frac{\epsilon(\tau\theta - \mu)}{(1 - \epsilon)} \right]^\epsilon$ from eq 9. Differentiating g_h w.r.t θ , we get

$$\frac{\partial g_h}{\partial \theta} = \left[\frac{\epsilon(\tau\theta - \mu)}{(1 - \epsilon)} \right]^\epsilon \cdot \frac{\epsilon\tau}{\tau\theta - \mu} = \frac{(1 + g_h)\epsilon\tau}{\tau\theta - \mu}$$

Substituting this into eq A10, we get

$$\frac{\partial g_H}{\partial \theta} = (1 + g_H) \frac{-\mu\beta_2(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^2} + (1 + g_h) \frac{\epsilon\tau}{\tau\theta - \mu} * n$$

Substituting for n ,

$$\begin{aligned} &= (1 + g_h) \left[\frac{\epsilon\tau\beta_2\theta(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^2} - \frac{\mu\beta_2(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^2} \right] \\ &= (1 + g_h)[\epsilon\tau\theta - \mu] \frac{\beta_2(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^2} \end{aligned}$$

Since it is given that $\theta > \frac{\mu}{\tau\epsilon}$, we have $\frac{\partial g_H}{\partial \theta} > 0$.

This proves proposition 2.

Appendix A3 : Proof of Proposition 3

From eq 43, the expression of growth rate of per capita income is given by:

$$g_y = (1 + g_h)^{\frac{\lambda}{1-\phi}} n^{\frac{\lambda+\phi-1}{1-\phi}}$$

Taking log on both sides

$$\log(1 + g_y) = \frac{\lambda}{(1 - \phi)} \log(1 + g_h) + \frac{\lambda + \phi - 1}{1 - \phi} \log n$$

Differentiating w.r.t θ , we get

$$\frac{1}{1+g_y} \frac{\partial g_y}{\partial \theta} = \frac{\lambda}{1-\phi(1+g_h)} \frac{\partial g_h}{\partial \theta} + \frac{\lambda+\phi-1}{1-\phi(n)} \frac{\partial n}{\partial \theta}$$

Substituting for n from eq (9) and $\frac{\partial g_h}{\partial \theta}$ from eq ??, we derive

$$\frac{\partial g_y}{\partial \theta} = (1+g_y) \left[\frac{\lambda}{1-\phi} \frac{\epsilon \tau}{\tau \theta - \mu} + \frac{(1-\lambda-\phi)}{1-\phi} \frac{\mu}{\theta(\tau \theta - \mu)} \right]$$

Since it is given that $\theta > \frac{\mu}{\tau \epsilon}$ and $0 < \phi \leq 1$ and $0 < \lambda \leq 1$, we have $\frac{\partial g_y}{\partial \theta} > 0$. Next, we differentiate 43 w.r.t ϵ ,

$$\frac{1}{1+g_y} \frac{\partial g_y}{\partial \epsilon} = \frac{\lambda}{1-\phi(1+g_h)} \frac{\partial g_h}{\partial \epsilon} + \frac{\lambda+\phi-1}{1-\phi(n)} \frac{\partial n}{\partial \epsilon}$$

We have already derived that

$$\frac{1}{1+g_h} \frac{\partial g_h}{\partial \epsilon} = 1 + \log \frac{\epsilon}{(1-\epsilon)} + \log(\tau \theta - \mu) + \frac{\epsilon}{1-\epsilon}$$

Also, from Lemma 2, we have $\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2 \theta}{(1+\beta_1+\beta_2)(\tau \theta - \mu)}$. Substituting for n , $\frac{\partial n_t}{\partial \epsilon}$ and $\frac{1}{1+g_h} \frac{\partial g_h}{\partial \epsilon}$ and simplifying, we get

$$\frac{\partial g_y}{\partial \epsilon} = \frac{\lambda(1+g_y)}{1-\phi} \left[1 + \frac{\epsilon}{1-\epsilon} + \log(\tau \theta - \mu) + \log \frac{\epsilon}{1-\epsilon} \right] + \frac{(1+g_y)(1-\lambda-\phi)}{(1-\phi)(1-\epsilon)}$$

Since it is given that $\theta > \frac{\mu}{\tau \epsilon}$ and $0 < \phi \leq 1$ and $0 < \lambda \leq 1$, we have $\frac{\partial g_y}{\partial \epsilon} > 0$. This proves proposition 3.

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